Technical Notes

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Induced Drag of a Slender Wing in a Nonuniform Stream

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Introduction

IN Ref. 1 a method was developed for calculating the pressure distribution and lift of a slender wing in a nonuniform parallel stream whose velocity varies in the vertical direction. The induced drag was not considered in Ref. 1; thus the evaluation of the induced drag is the subject of this Note.

To obtain the induced drag, we find the leading-edge suction forces and subtract their resultant from the streamwise component of the pressure force on the wing. Since the leading-edge suction force has a purely local character, it can be derived from two-dimensional theory. The derivation shows that the leading-edge suction force in a nonuniform stream is related to the singularity of velocity in the same way as in a uniform stream. Using this relation, the induced drag is calculated from the solutions of Ref. 1. Numerical results for several stream velocity profiles show how the induced drag varies with the velocity ratio of the stream and with the ratio of wing span to the vertical extent of stream nonuniformity.

Two-Dimensional Flow Analysis

Since the local flow near the leading edge of a wing is effectively two-dimensional in planes normal to the edge, the suction force on the leading edge can be found by considering two-dimensional flow over an airfoil.[2] For a nonuniform stream we can use the airfoil theory given by Weissinger.[3] The theory assumes small perturbations of the nonuniform stream and employs linearized Euler equations of flow, as is also done in Ref. 1. Weissinger obtained an integral equation for the vorticity distribution on the airfoil and investigated the lift and drag. He showed that the D'Alembert paradox remains valid for airfoils in nonuniform stream, i.e., the airfoil has zero induced drag. It follows that the leading-edge suction force of an airfoil balances the streamwise component of the pressure force. Expressing the pressure in terms of vorticity using the linearized x-momentum equation, we have

$$T' = \rho U(0) \int_0^c \gamma(\xi)\alpha(\xi)\,\mathrm{d}\xi \tag{1}$$

Here T' denotes leading-edge suction force of a two-dimensional airfoil, $\gamma(\xi)$ the vorticity distribution on the airfoil, $\alpha(\xi)$ the local angle of attack, ξ distance from the leading edge, c the chord, ρ the air density, and $U(0)$ the stream velocity at the airfoil plane $z=0$.

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The integral equation obtained by Weissinger[3] can be stated in the form

$$\int_0^c (\xi-\xi_1)^{-1}\gamma(\xi_1)\,\mathrm{d}\xi_1 = -2\pi U(0)\left[\alpha(\xi)+\omega(\xi)\right] \tag{2}$$

$$\omega(\xi) = \int_0^c Q(\xi-\xi_1)\gamma(\xi_1)\,\mathrm{d}\xi_1 \tag{3}$$

where the term $\omega(\xi)$ is due to the stream nonuniformity. The kernel Q of this term is a continuous odd function which depends on the stream velocity profile $U(z)$ only. In Eq. (2) the integral is defined as its Cauchy principal value.

Eliminating $\alpha(\xi)$ from Eq. (1) by means of Eqs. (2) and (3) gives

$$T' = -\frac{\rho}{2\pi}\int_0^c \gamma(\xi)\,\mathrm{d}\xi \int_0^c (\xi-\xi_1)^{-1}\gamma(\xi_1)\,\mathrm{d}\xi_1$$

$$-\rho U(0)\int_0^c\int_0^c \gamma(\xi)Q(\xi-\xi_1)\gamma(\xi_1)\,\mathrm{d}\xi_1\,\mathrm{d}\xi \tag{4}$$

and the last term vanishes since Q is an odd continuous function. Thus we have

$$T' = -\frac{\rho}{2\pi}\int_0^c \gamma(\xi)\,\mathrm{d}\xi \int_0^c (\xi-\xi_1)^{-1}\gamma(\xi_1)\,\mathrm{d}\xi_1 \tag{5}$$

This shows that the leading-edge suction force in nonuniform stream is related to the vorticity distribution $\gamma(\xi)$ on the airfoil in the same way as for a uniform stream. A simple relationship, well known in the case of uniform stream, can be deduced from Eq. (5) as follows. From the theory of singular integral equations[4] we know that the vorticity γ must behave like $\xi^{-1/2}$ near the leading edge, since it is a solution of Eq. (2) and satisfies the Kutta condition $\gamma(c)=0$. The integral in Eq. (5) can then be evaluated by setting $\gamma(\xi)=\xi^{-1/2}g(\xi)$ and expanding g in a Fourier series. The result is

$$T' = (\pi/4)\rho G^2 \tag{6a}$$

$$G = \lim_{\xi\to 0} \xi^{1/2}\gamma(\xi) \tag{6b}$$

Slender Wing in Nonuniform Stream

We consider now a slender wing set at an angle of attack in a parallel nonuniform stream whose velocity $U(z)$ varies in the vertical direction. The x, y and z axes are taken in the streamwise, spanwise, and vertical directions, respectively. The wing is assumed to have zero thickness and is located near the $z=0$ plane. The notation is the same as in Ref. 1.

For three-dimensional wings, the flow near the leading edge is effectively two-dimensional in planes normal to the edge, since the velocity component parallel to the edge is continuous and does not contribute to the suction force. Consequently, Eqs. (6) hold locally along the leading edge of a three-dimensional wing, with T', γ, and ξ taken in planes normal to the edge. It follows that the leading-edge suction force T of a pointed slender wing, between the apex and the spanwise section x, is obtained from

$$T = 2\int_0^x s'(x_1)T'(x_1)\,\mathrm{d}x_1 = \frac{\pi}{2}\rho\int_0^x s'(x_1)G^2(x_1)\,\mathrm{d}x_1 \tag{7}$$

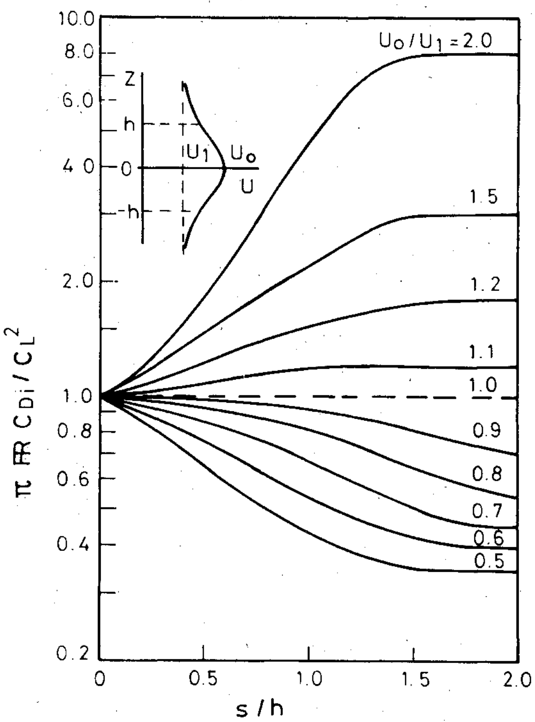

Fig. 1 Induced drag in jet and wake streams.

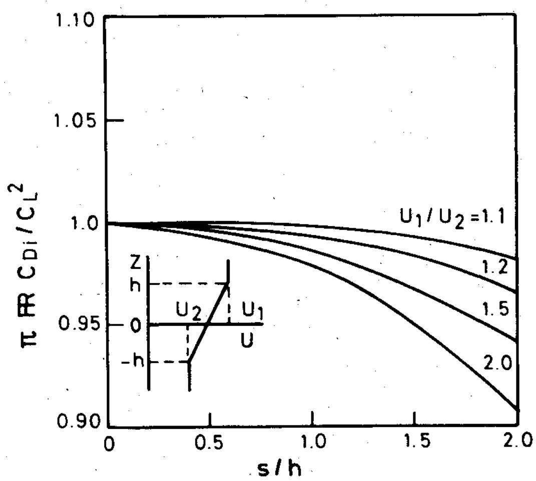

Fig. 2 Induced drag in linearly sheared stream.

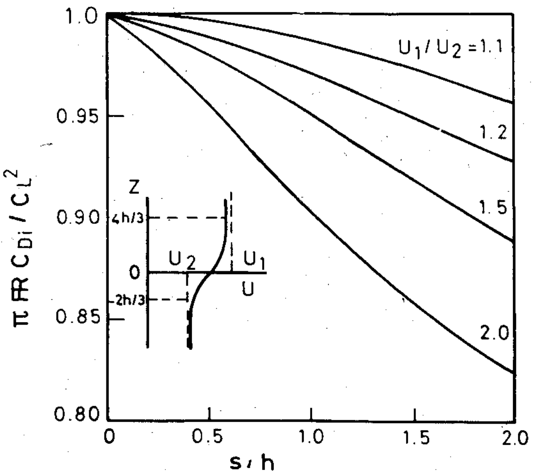

Fig. 3 Induced drag in nonlinear sheared stream.

Here x denotes the chordwise coordinate, $s(x)$ the local semispan, and G is found from the vorticity or pressure distribution near the leading edge using Eq. (6b).

In Ref. 1 a solution was given for the pressure load on a slender wing in nonuniform stream. The solution was expressed in terms of the lift distribution function

$$L(x,y) = \frac{2}{\rho U^2(0)} \int_{x_\ell(y)}^{x} [p(x_1,y,-0) - p(x_1,y,+0)]\mathrm{d}x_1 \qquad (8)$$

where p is the pressure perturbation, y the spanwise coordinate, and $x_\ell(y)$ the leading edge. The lift distribution was expanded spanwise in a Fourier series

$$L(x,y) = s(x) \sum_{m=1}^{\infty} L_m(x)\sin(m\phi) \qquad (9)$$

where

$$y = s(x)\cos\phi \qquad (10)$$

The coefficients L_m were determined in Ref. 1 by solving the integral equation of a slender wing in nonuniform stream.

The vorticity on the wing is related to the lift distribution L through the linearized momentum equations. For a slender wing we have

$$\gamma(x,y) = \frac{1}{2} U(0) \frac{\partial L(x,y)}{\partial y} \qquad (11)$$

Using Eqs. (6b) and (9-11) we can express the function $G(x)$ in terms of the coefficients $L_m(x)$ as follows

$$G(x) = \frac{1}{2\sqrt{2}} U(0)\sqrt{s(x)} \sum_{m=1}^{\infty} mL_m(x) \qquad (12)$$

The leading-edge suction force of the slender wing in nonuniform stream is now obtained from Eqs. (7) and (12), and the result is

$$T = \frac{\pi}{16}\rho U^2(0) \int_0^x s(x_1)s'(x_1)\left[\sum_{m=1}^{\infty} mL_m(x_1)\right]^2 \mathrm{d}x_1 \qquad (13)$$

The streamwise component of the pressure force on the wing P_x can be found directly from the lift distribution function $L(x,y)$ and the angle of attack $\alpha(x,y)$. For a plane slender wing

$$P_x = (\pi/4)\rho U^2(0)s^2(x)L_1(x)\alpha \qquad (14)$$

Finally, the induced drag D_i is obtained by subtracting

$$D_i = P_x - T \qquad (15)$$

Numerical Results

Values of the induced drag were calculated for a plane slender wing and several stream velocity profiles representing flight in a jet, a wake, and in sheared wind. The stream profiles are the same as those for which the lift distributions were computed in Ref. 1, and they are sketched here in inserts of Figs. 1-3. The profiles are characterized by two values of velocity (the maximum and minimum) and a parameter h related to the vertical extent (height) of stream nonuniformity.

The calculated values of the induced drag factor

$$k = \pi AR C_{Di}/C_L^2 \qquad (16)$$

are shown in Figs. 1-3 as functions of the velocity ratio U_0/U_1 or U_1/U_2 across the stream and the nonuniformity scale ratio s/h. Here C_L and C_{Di} are the lift and induced drag coefficients based on the local stream velocity $U(0)$ at the wing plane, and s is the wing semispan. The factor k does not depend on the angle of attack or the lift coefficient, and its value in a uniform stream is $k=1$. Since the coefficients C_L and C_{Di} are based on $U(0)$, they will tend to their uniform stream values as the scale ratio is decreased.

It is found that the effects of stream nonuniformity on the induced drag are significant in a wide range of the parameters. The variations of the drag factor k with the velocity ratio U_0/U_1 or U_1/U_2 become stronger as the scale ratio s/h increases. For the jet and wake streams the effects are much larger than for the linearly sheared stream; this is due to the dependence of the solution for lift distribution on the second derivative of stream profile $U''(0)$, which was discussed in Ref. 1.

References

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Simple and Accurate Calculation of Supersonic Nozzle Contour

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Introduction

THE method of characteristics is commonly used to design a supersonic nozzle.[1] This method is widely applied to large nozzles where the boundary-layer displacement thickness is small compared to the nonviscous flow. As the nozzle becomes larger and the Mach number higher, one should use more characteristics lines in order to achieve an accurate supersonic profile. Such a profile is necessary to obtain a uniform flowfield at the exit of the nozzle in the supersonic test chamber. A simple method that considerably increases the accuracy of the solution for a given number of characteristics lines is described in this paper.

The Method

Let us assume a straight sonic line at the throat of a supersonic nozzle, perpendicular to the flow direction. To calculate the flow in the supersonic region, the characteristics lines are taken as straight segments between two grid points.[1] There are few procedures that take the curvature of the lines into consideration.[1-3] However, these procedures give systematical error in the calculated supersonic nozzle wall profile that is usually overcome by increasing the number of characteristics lines.[2] One way to check the accuracy of the calculation is to compare the final area ratio of the supersonic nozzle that is obtained by the method of characteristics to the one-dimensional area ratio for isentropic flow with the same specific heat and final Mach number. These two solutions should coincide since the flow is assumed to be uniform and the cross-sectional area perpendicular to the flow direction at both the throat and the nozzle exit. It is suggested that the same procedure be applied for each segment of the expansion waves prior to the construction of the grid in the characteristics calculation, as opposed to the procedure of determining the contour by streamlines for a given grid.[4] Following this method, one gets a supersonic nozzle wall boundary contour that is an envelope of the accurate one.

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To explain the method, we consider the simple case of isentropic expansion flow near a convex corner, as shown in Fig. 1. Assuming that the flowfield outside the convex corner region is supersonic and uniform at Mach numbers M_1 and M_2 before and after the curve, respectively, the expansion waves are straight. Since there is no characteristics length to define a scale in the configuration perpendicular to the streamlines, the flow parameters must be constant along Mach line "rays" that are initiated at the corner. For each Mach number M, one defines a Mach line with an angle μ with respect to the flow direction at a particular point of the flow: $\mu = \sin^{-1}(1/M)$. Let μ_1 and μ_2 be the Mach angles corresponding to the Mach numbers M_1 and M_2 at the boundaries of the expansion fan, and θ the angle change in the flow direction. Using the method of waves it is desired to represent the expansion fan by a single Mach line so that the incoming and outgoing streamlines will be straight. However, none of the μ_1 and μ_2, or the averaging procedures used,[3] can keep the streamlines outside the expansion fan unchanged. The relation between the Prandtl-Meyer functions ν_1 and ν_2 that correspond to M_1 and M_2, respectively, and θ, in simple isentropic turns, implies that $\nu_2 - \nu_2 = \theta$ where

$$\nu(M) = [(\gamma+1)/(\gamma-1)]^{1/2}\tan^{-1}$$
$$\times [(\gamma-1)(M^2-1)/(\gamma+1)]^{1/2} - \tan^{-1}(M^2-1)^{1/2}$$

where θ is the curve angle, the angle between the direction of a streamline before and after the curve region, and γ the ratio of specific heats.

Since θ, γ, and M_1 are defined, one can calculate M_2. On the other hand, since the flow is isentropic and since y_1 and y_2 are measured perpendicular to the flow direction, one can use the continuity equation for the flow that is limited between two streamlines, as is done in the one-dimensional flow. Thus,

$$\frac{y_2}{y_1} = \frac{M_1}{M_2}\left[\left(1+\frac{\gamma-1}{2}M_2^2\right)\Big/\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Therefore, for a given M_1, M_2, γ, and θ, y_2 is defined for each y_1. The streamlines are curved inside the expansion fan and straight outside. The next step is to find the intersection of the two straight lines that corresponds to the same streamline, before and after the expansion fan, inside the fan region. These two lines inside the expansion fan will be called the streamline envelope.

Since the streamlines outside the expansion fan are straight and parallel, their intersections define a straight line. An angle ω is defined between the new "Mach line" and the direction of the flow before it enters the expansion fan. ω can be found from the expression

$$y_1/y_2 = \sin(\omega)/\sin(\omega+\theta)$$

The new "Mach line" represents the flow inside the expansion fan, keeping the streamlines unchanged outside the fan region, and defines straight streamlines envelopes inside the fan. This procedure can also be applied to the wall nozzle boundary, since it also defines a streamline.

To calculate a contour of a supersonic nozzle wall boundary, a regular characteristics method should be followed.[3] In this procedure one divides the deflection angle into equal segments. However, instead of taking the inclination of each wave at angle μ, the new "Mach line" (at angle ω) should be taken. It keeps both the positions and slopes of the streamlines on the boundaries of each segment unchanged. Figure 2 describes the envelope of a supersonic nozzle wall boundary produced by one and two characteristics calculations for a sharp corner nozzle. The contours are so